

beauty & truth
MATH

Mathematics in a
Charlotte Mason Education

PRACTICAL GEOMETRY

SAMPLE

YEAR 1 • TERM 1

SAMPLE

Beauty & Truth Math

- Mathematics in a Charlotte Mason Education -

PRACTICAL GEOMETRY

YEAR 1 • TERM 1

Used in conjunction with

LESSONS IN EXPERIMENTAL AND PRACTICAL GEOMETRY

by H.S. Hall and F.H. Stevens

PRACTICAL GEOMETRY • YEAR 1 • TERM 1

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We turn from the sciences of facts as we see them to the science of facts as they must be. Truth is the key-note and core of mathematics. There is no "nearly right" or "probably is so" or "certainly may be" about $2+2=4$. Logic, the putting of two and two mentally and inevitably together, and truth in all her majesty and tidiness are to be the mental acquisitions gained from arithmetic, Euclid, and algebra.

(Pennethorne, The Parents' Review, 1899, Vol. 10, p. 559)

The application of algebra to curves, etc., brings us to its meeting-point with geometry. Nothing can help children better to understand that abstract logical reasoning is not unreal than the mental discipline of going from figure to proof, and proof to figure, in what we term "propositions." Geometry trains the mind to severe reasoning, the hand to absolute accuracy, and it lies at the root base of many important and honourable professions, which is a real, though utilitarian, reason why we should teach it.

Geometry is especially remarkable in that it converts each idea it gives into a habit of mind or action in construction, and a base for the next idea to rise upon. Thus, when we give a lesson upon a proposition, we make clear the idea (equality perhaps of two triangles) to be conveyed, and then help the pupil to discover the logical as well as the obvious proof of its reality, and never make the children learn by heart without comprehension a chatter of ABC=Q.E.D.

(Pennethorne, The Parents' Review, 1899, Vol. 10, pp. 559-560)

Knowledge is valued by our society as a source of delight and of power, and as enabling the character and the intellect to develop, and, in this regard, the mode in which the knowledge is acquired is all-important.

Take geometry. Foreign nations laugh at us for still sticking to the old methods of Euclid. Euclid says: These are the facts and here are the reasons for it.

Teaching in the spirit of the P.N.E.U. would say: Here are the lines and the angles, what can you find out about them?

(Nesbitt, The Parents' Review, 1904, Vol. 1, pp. 96, 98)

SAMPLE

WELCOME

Thank you for purchasing this guide! We are humbled and honored by your support. Please read through this introduction carefully. Understanding our approach is vital to maximizing the benefits of each guide.

THE VISION

Beauty & Truth Math exists to assist students AND teachers in the realm of mathematics in a Charlotte Mason education. Reading the scripted lessons and checking your students' answers is possible. However, this keeps the teacher from being an engaged and involved partner in the learning process.

These lessons are written with the idea that the teacher will work with the students, ask questions, have discussions, and monitor progress. Each lesson is an opportunity for building relationships between you, your students, and the Lord. Please make the most of this time together, walking beside your students in exploring and understanding mathematical ideas.

We thoroughly believe that math done completely in isolation misses opportunities to make deep connections. Just like a foreign language needs to be communicated and spoken to make connections, math is its own language with its own big ideas that are best learned through discussion.

You are working with *living*, born persons; we aim to provide a *living* education. *Living* involves changes and adaptations. These lessons are guides and servants, not masters you must follow. Please use the Spirit's wisdom when discerning what you should modify, skip altogether, push forward on, or slow down on as you and your students are on this journey.

AN IMPORTANT NOTE

The Beauty and Truth Math curriculum is a marriage of spiral and mastery. The weekly review allows the students to continue working on challenging ideas without plateauing. We also teach the reasonableness of each math concept and then recap it each year to help build conceptual understanding.

We never want students to get bogged down in an idea they have not fully mastered. Please keep moving forward even if memorization has not occurred. Your student's relationship with the Lord, you, and math are more important than memorizing a formula!



CONTACTING US

We welcome feedback and questions! For general inquiries, please email us at contact@beautyandtruthmath.com.

For questions about this guide's content, please email Emily Al-Khatib at emily@beautyandtruthmath.com. We have done our best to remove all typos from these guides. However, if you find any mistakes, please forgive us and let us know.

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These guides have been a labor of love. Please respect our hard work and do not share any content and links that are not publicly available on our site.

WEBSITE LINKS DISCLAIMER

PLEASE PREVIEW LINKS BEFORE USING! While we have done our best to ensure all sites we link to are appropriate, we do not have any control over their changes.

We are thankful for the free resources other sites make available and want to support them whenever possible. We link directly to their pages as they generate revenue through site traffic.

In many cases, the linked pages will provide multiple worksheet options. Most of the time, we will specify which worksheet you will need in the guide. Sometimes, you must choose the worksheet, which the guide will also say.

It is the teacher's final responsibility to ensure the content is age-appropriate for the lessons. Please email us at contact@beautyandtruthmath.com to report broken links.



READY, SET, GO!

"Putting in the work up front to make the school days run easy."

We have created three folders to easily access the teacher help documents and printables included in this guide. Their unique QR codes and links are included in multiple places in this introduction and are shown here for easy identification.

We will walk you through using these linked folders in the following few pages, so please don't worry about viewing them now. This page is simply an introduction to them.

Important Teacher Helps – This folder contains helpful resources to assist and support you as you implement math in a Charlotte Mason education. It includes the following documents:

- A CHARLOTTE MASON MATH EDUCATION outlines a vision for a Charlotte Mason math education.
- ALL ABOUT THE GUIDES (PG1) has everything you need to know about the guide's setup.
- LEPG SOLUTIONS provides the constructions to assigned problems from *Lessons in Experimental and Practical Geometry*.
- PROGRAMME EXAM ANSWER KEY provides the construction answers to the P.U.S. Programme Exam questions.
- SUPPORT VIDEOS LIST is a compiled, linked list of support videos in this guide.
- ADDITIONAL SUPPLEMENTAL RESOURCES provide extra teacher support.






Cardstock Printables – This folder contains all resources that need to be printed on cardstock, as these will be used with your students multiple times throughout the lessons.



Printables – This folder contains all of the consumable printables for your students.



GET READY!

- **SEE** the **Materials Needed** section in this guide to determine what materials you have and still need to purchase.
- **BUY** *Lessons in Experimental and Practical Geometry* (1911) in physical and/or digital form and the geometry toolkit. **Our Favorites** page links the textbook and other recommendations we have compiled to help you prepare and organize your materials. 
- **PRINT ALL OF THE DOCUMENTS** in the **Cardstock Printables** folder. You will use these documents multiple times, so we recommend using cardstock paper. 
- **PRINT ALL OF THE DOCUMENTS** in the **Printables** folder. 
- **DECIDE** if you will print the guide or use it on a screen.



GET SET!

READ THROUGH THE FOLLOWING IMPORTANT TEACHER HELPS:

- A CHARLOTTE MASON MATH EDUCATION
- ALL ABOUT THE GUIDES (PG1)



Prepare your materials. There is no one right way to do this! The following list is simply a compilation of ideas Beauty & Truth Math users have found helpful.

- **Create a student math notebook for each student.**
 - Fill it with both graph and unlined, blank paper. In general, we recommend $\frac{1}{4}$ " squares for the graph paper. Some students may need larger squares based on their writing ability.
 - Create sections in the notebook for daily assignments and a reference section. Decide with your student how to order these. If applicable, create different sections for the different streams of math.
 - Decide if you will have your student write headings for each assignment. Information such as the date and page number are great things to include. Writing the problem number and showing the final answer, either with a box or a circle around it, is also strongly encouraged.
- **Put together a teacher math notebook for yourself.**
 - Create sections for your personal calendar, the lessons from the guides, printable & supplementary resources, exams, notes, etc.
- **Find a place for the Cardstock Printables.**
 - Store these in a folder or pocket of your teacher's notebook or inside an accordion file folder. The goal is to keep them accessible and in good condition since you will use them often.
- **Use tabs to label and easily find what you need!**
 - In the *Lessons in Experimental and Practical Geometry* text, tab the current lesson and the corresponding answer key section in the back of the book.
 - If you choose to print the LEPG Solutions document, place a tab where you are at.
- **Have individual containers for each of your student's supplies.**



GO!

Any author of math textbooks or guides will tell you that we write to accommodate as many students as possible and provide more than is needed. You have complete freedom not only to modify the lessons but also to adjust the number of problems assigned to meet the needs of your students.

Each week, you will need to do the following:

- Look over the new lesson to be covered with your student. Understand the big ideas and objectives.
 - If there are constructions that you are unfamiliar with, take a few minutes and go through the steps to complete them on your own.
- Take the Beauty & Truth Math Guide Vow – I do solemnly promise that I will remember and implement the following statements:
 - I have permission from Charlotte Mason and the authors of these lessons to adjust or modify any lesson, at any time, to provide a living education to my unique, born persons.
 - I have permission from Charlotte Mason and the authors of these lessons to assign fewer problems than written in the lessons to provide a living education to my unique, born persons.
 - I have permission from Charlotte Mason and the authors of these lessons to assign more problems than written in the lessons to provide a living education to my unique, born persons.

“..the educator has to deal with a self-acting, self-developing being, and his business is to guide, and assist in, the production of the latent good in that being, the dissipation of the latent evil, the preparation of the child to take his place in the world at his best, with every capacity for good that is in him developed into a power.” (Mason, Vol. 1, p. 9)

- Pray for joy and wisdom as you explore mathematical truths with your students each day. Now dive right into using the lessons, confident that the Lord is with you and for you!



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ALL ABOUT THE TERM

There is no one-size-fits-all way to teach math using the Charlotte Mason method. Our guides are one option for teachers to use. We have created them to be adaptable to each unique student, both in the big picture and in the guides' details.

We have designed our curriculum to imitate the math streams used in Charlotte Mason's schools. Students have several options for the tracks and combinations of these streams. For more information, see our [Scope & Sequence](#) page on our website.



Our [Guides' Big Ideas](#) page shows the main ideas throughout the years.



TERM OVERVIEW

This term, we introduce students to the formal study of geometry, including foundational vocabulary. They learn to measure length in inches, tenths of an inch, centimeters, and millimeters. In the second half of the term, students investigate the properties of circles, including the intersection of two circles.

The **maximum** lesson time for students in Practical Geometry, Year 1 is 30 minutes, and lessons occur once a week.

EVERYDAY & SPECIAL MATERIALS

We assume students will always have their pencil, math notebook with grid and blank paper, the geometry toolkit, ruler, and a large eraser handy for lesson time. Any additional materials beyond these items are listed in the Special Materials Needed section.

CARDSTOCK PRINTABLES VS. PRINTABLES

The teacher must prepare all cardstock printables before the term begins. The cardstock printables are listed as special materials, but links are not provided. Links for the Printables Folder are always provided in the special materials.



PAGE NUMBERS

Page numbers are referenced throughout the guide. These are taken from Yesterday's Classics' edition of *Lessons in Experimental and Practical Geometry* (1911). Other editions are available. However, the guide will be harder to navigate because the page numbers will not match.



THE DETAILS MATTER

In Arithmetic Year 2, students learned about a costly mistake NASA engineers made in 1999. Throughout this guide, references will be made to that story to remind students to label their answers. If your students are unaware of the Mars Climate Orbiter disaster, please read *The Details Matter (NASA's Costly Mistake)* to them.



REFERENCE PAGES

Students use the Geometry Vocabulary, Geometry Observations, and Geometry Symbols Pages throughout the term. Print these on cardstock paper to help with durability, as the student will use them frequently over the two years of practical geometry. Please keep these pages throughout Practical Geometry 1 and 2, allowing students to continuously add to them. If needed, extra copies of the reference pages can be found [here](#).



These pages may be referenced to jog the students' memories about previously learned ideas. Students may add to them at any point as they continue to grow in their knowledge and understanding of mathematics.

REFERENCE PAGES OVERVIEW AND ANSWER KEYS

The Printables folder includes *Reference Pages Overview and Answer Keys*. This document provides an overview of all the reference pages used throughout both years of Practical Geometry, showing the culmination of students' two years of hard work. Please print this document for your reference.



INDEPENDENT STUDENT WORK

The Printables folder includes *Independent Student Work*. This document has all the Student Response sections that can be done without the teacher's help, so it excludes the answers.



PRACTICAL GEOMETRY SUPPORT VIDEOS

There are support videos to assist students and teachers use geometry tools and complete geometric constructions. Before beginning this course, take a few minutes to watch [Getting to Know the Tools of Geometry](#).



Every foundational construction in *Lessons in Experimental and Practical Geometry* (1911) is titled “Problem.” For example, Problem 1 is in Ch. IV (p. 23). Videos are provided for each on the Beauty & Truth Math Practical Geometry YouTube Channel. When these are referenced in the guide, a link and QR code will be provided.

SOLUTIONS

All numerical answers are provided in the back of *Lessons in Experimental and Practical Geometry*, but no constructions or drawings are given. If you are looking for an answer and do not see it in the back of the book, please see [LEPG Solutions](#). There are solutions for every problem in the *Lessons in Experimental and Practical Geometry* text. Each one is labeled by chapter and exercise number. Consider printing this document and placing it in your teacher’s notebook for easy reference.



The answers to the Programme exam questions are given in the [PG.Y1.T1 Exam Answer Key](#).

LESSON MODIFICATIONS

The Practical Geometry Guides **do not** have the moving van symbol as found in other guides. Reference page assignments can always be shortened or assigned as ongoing assignments, and students can pick up where they left off in the next lesson time if an assignment is not completed. Occasionally, arithmetic review/catch-up days can be used to complete a geometry lesson.



MATERIALS NEEDED

Our Favorites provides links to *Lessons in Experimental and Practical Geometry* as well as other recommendations that may be helpful to you in preparing and organizing your materials.



Textbook

Lessons in Experimental and Practical Geometry (1911) from Yesterday's Classics

Cardstock Printables

- PG.Y1.T1 Reference Pages



Everyday Materials

- Notebook with Graph & Blank Paper
- Pencil
- Geometry Toolkit (includes a compass, dividers, protractor, and set squares)
- Ruler (with markings of inches in tenths and sixteenths of an inch and centimeters and millimeters)
- Large Eraser

Printables

- PG.Reference Pages Overview & Answer Keys (Optional)
- PG.Y1.T1 Independent Student Work (Optional)



SPECIAL MATERIALS

- Circular Object (bicycle wheel, embroidery hoop, hula hoop, etc.)
- Colored Pencils or Markers
- Internet Access
- Paper Clip
- Piece of Cardstock Paper

- Scissors
- Templates #1, #2, and/or #3 (Choose what you would like to print.)
- Two Sharpened Pencils
- Wooden Solids



• Introduction to Geometry •

PG1. T1. W1.

SUBJECT

Geometry



RESOURCES USED

Lessons in Experimental and Practical Geometry (Ch. 1)

OBJECTIVES

Students will be able to describe a surface, face, line, and point.


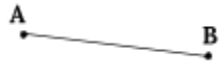
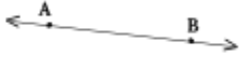
SPECIAL MATERIALS NEEDED

Internet Access, Wooden Solids, Geometry Vocabulary Page

THE PLAN

1. This year, we will begin studying a new type of mathematics called geometry. Let's break that word down a bit to think about its meaning.
 - a. What do you think the first part of *geo-* means? Maybe you could think of another subject that you study that sounds similar. (Geography) *Geo* means the earth. In geometry and geography, you study the Earth from different perspectives.
 - b. What about the second part of geometry, *-metry*? It comes from the Greek *metron*, which means to measure.
 - c. When we study geometry, we explore how to measure the Earth.
2. Watch a [brief history of geometry](#) (2:40).
3. There are other branches of geometry, but we will be studying plane or flat geometry because we will be learning about things that can be drawn on flat paper or a plane, like Egyptian fields. We will look at the world around us. Are your surroundings plane or flat? (No) They aren't flat, but we will practice taking things that aren't flat and imagining that they are flat or using the flat parts to describe the whole object. Let's see what that looks like now.
4. First, can you name the shapes that you see? *Help the student name the cube, sphere, cylinder, cone, and pyramid, and use those solids for the next steps. Put up the rest of the solids, as we will not use them now.*
5. Look at the cube and the sphere.
 - a. Are these objects flat or plane? (No) But let's look at the flat or plane things on them.
 - b. Can you explain what a surface is? (The surface of objects is the outside of them.)
 - c. How could you describe the **surface** of these solids? (The outside of the solid)



- d. What is the difference between the cube's and sphere's surfaces? (The cube's surface is in several pieces, and the sphere's surface is in just one piece.)
 - e. Pick up the sphere, cone, and cylinder. Looking at each, how many surface pieces are there? (The sphere has one, the cone has two, and the cylinder has three.)
 - f. Can you imagine that you cut these solids open and lay them out on the table, then describe those pieces of the surfaces? *Draw out the idea that some are straight and some are rounded.*
 - g. How would you describe what the surface of a solid is? *Draw the idea of the outside of the solid - what can be seen and touched.*
6. Now, pick up the cube.
- a. Tell me how many pieces the surface is. (6 pieces)
 - b. Each of these pieces or sides is called a **face**. So how many faces does a cube have? (6 faces)
 - c. How would you describe what the face of a solid is? *Draw from the student that the face is a part of the surface of the solid.*
7. Let's continue looking at the cube but focus on what happens when these faces come together.
- a. What do you notice about where the two faces meet? (They meet on an edge.) This edge represents a **line**.
 - b. Where do three faces intersect? (They intersect in a corner.) This represents a **point**.
 - c. Look at some other solids and find examples of lines and points. Find some examples of points and lines around the room.
8. In geometry, a **point** is considered a location in space. We represent it with a small dot and name or label it with an uppercase print letter.
- a. Can you draw and label point A on your dry-erase board? Point B? 
 - b. Now we are going to connect those points. Draw at least three different ways to connect those two points. (i.e., loops, curved, zigzag, and straight lines)
 - c. Which one of those ways is the shortest way of connecting A and B? (The straight line)
 - d. What is this figure called? (A line) 
9. The Greek mathematician, Euclid, known as the "father of geometry," referred to a **line** as "breadthless length." What do you think he meant by this? A true line has only one dimension, and believe it or not, exists only in our imaginations. It is measured only in length, has no width, and has no stopping point in either direction. Though when we draw a line, our pencil or pen gives it width; otherwise, we wouldn't be able to see it! So our drawing is a *representation* of a line.
10. Go back and extend the line past points A and B. Then draw arrows at both ends to show that the line does not end. 



11. We name a line according to any two points it passes through. So we say “line AB” and write it as \overleftrightarrow{AB} .

STUDENT RESPONSE

- Throughout this year, you will continually add to a special reference page called the Geometry Vocabulary Page. Every time you learn a new word, you will write it, explain it in your own words, and draw a picture to illustrate it. Let's try it now. As time permits, add the following words to the Geometry Vocabulary Page. Include a definition and an illustration of each.
 - Surface
 - Face
 - Point
 - Line
- Optional Afternoon Activity: Be a Rope-Stretcher! *The purpose of this activity is not supposed to show the students any theorem or proof. It is only supposed to help them to imagine how things might be measured without the aid of modern measuring tools and to show what led to the development of geometry.*
 - Take a string that is two feet long.
 - See if you can tie knots in it at 12 equal increments without the help of a ruler or measuring tape. How hard is that? Can you see why tools of standard measurements, such as the foot ruler, yardstick, etc.) were developed and their importance?
 - If your student struggled with the knot-tying, please provide the string with pre-tied knots at this point. Try your hand at being a rope stretcher. What can you measure? How is this knotted rope helpful? Try to make a triangle. How could this shape be helpful? Can you measure your room using this rope? *Guide the student in imagining a rectangle by combining two triangles to see how the Egyptians measured the fields.*



• Straight and Curved Lines •

PG1. T1. W2.

SUBJECT

Geometry



RESOURCES USED

Lessons in Experimental and Practical Geometry (Ch. I)

OBJECTIVES

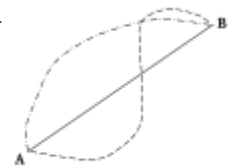
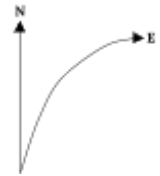
Students will be able to investigate the properties of lines.

SPECIAL MATERIALS NEEDED

Internet Access, Geometry Observations Page

THE PLAN

1. Tell me what you remember from our last geometry lesson. *Draw out the ideas of the history of geometry, surfaces and faces of solids, points, and lines.*
2. Let's consider some different situations that include points and lines. Find a blank page in your math notebook. In your math notebook, you will list what is true about points and lines, writing down your conclusions after each investigation. Some of these may seem a little strange because they are pretty obvious, but that's OK. You will have a foundation that will last throughout your whole geometry study for the rest of your life!
 - a. Think about taking two different walks from the same starting place. (Based on Ch. I: (i)).
 - i. Draw a diagram of a path that continually goes north.
 - ii. Draw another path that starts going north and then gradually moves to the east.
 - iii. Which one of these paths represents a straight line? What can you say about a straight line and its direction? (A straight line continues in the same direction.)
 - b. "In a field, there are two trees whose positions are marked by the letters A and B. Suppose you want to go from one tree to the other by the shortest way" (Ch. I: (ii)).
 - i. Draw your path.
 - ii. How many of these lines can there be? (Only one)
 - iii. What can you say about a line in this example? (There can be only one line with the shortest distance between two points.)



- c. “A strip of ground has been enclosed by two fences. One of these is straight; can the other fence be straight also?” (Ch. I: (iii)).
- Draw a picture of the two fences.
 - Can two straight lines enclose a space? (Two straight lines cannot enclose a space. One must be curved.)
 - What is the smallest number of straight lines needed to enclose a space? (Three straight lines are needed to enclose a space.)
- d. “We will draw a curve and call it \overleftrightarrow{AB} ; then we will [draw] a straight line \overleftrightarrow{CD} across it” (Ch. I: (iv)).
- Draw a picture.
 - How many times does the straight line cut the curved one? (Depending on the curved line, a straight line can cross it at 0, 1, or more points.)
- e. “Now take a straight line \overleftrightarrow{AB} , and [draw] another straight line \overleftrightarrow{CD} across it” (Ch. I: (iv)).
- Draw a picture.
 - Can \overleftrightarrow{CD} cross \overleftrightarrow{AB} at more than one point? (No)
 - What can you say about the intersection of two straight lines? (If two straight lines cross one another, then they can intersect at only one point.)
3. For more information about Euclid and to summarize some of the things we discussed last time, check out [What's the Point of Geometry?](#) (3:18).
- If you keep a timeline, add Euclid to it.



🔍 OBSERVATIONS

Read through your conclusions about straight lines. Compare them to the list on page 5. You have just made some important observations about the characteristics of straight lines! Great work today!

STUDENT RESPONSE

- Complete Ch. I: Ex. 1 and 2 in your math notebook.
- Throughout your geometry study, you get to keep track of your own Geometry Observations Page. This is where you can record your observations and discoveries because there will be many! You will be using this page for the next two years! To start, observations about straight lines you saw today are examples. Give this group of observations a title and write it in the box. Then draw an example of each one. *The titles for observations can be as simple and straightforward as possible. One possibility for this set of observations is “The Characteristics of Straight Lines,” but it is perfectly fine if your student wants to use something different.*